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Interframe Television Coding Using Gain and Displacement Compensation

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This paper presents algorithms for predicting luminance changes in successive television frames. The changes can result when objects in a TV scene move or when illumination varies. By a gradient search technique, which seeks to minimize a functional of the interframe prediction error, we estimate two parameters associated with these luminance changes—displacement and gain. Using the estimates of these parameters, we also develop, for interframe coding, adaptive predictors and a segmentor to determine which pels need to be transmitted. We describe several coder variations and compare them by computer simulations using three substantially different scene sequences. For these sequences, gain compensation with improved segmentation reduced the bit rate of a conditional replenishment encoder by 50.7, 11.1, and 39.3 percent. Displacement compensation reduced the bit rate by 61.0, 24.8, and 14.5 percent. Combined gain and displacement compensation reduced the bit rate by 63.4, 32.2, and 44.6 percent.

I. INTRODUCTION

Television signals contain a significant amount of frame-to-frame redundancy because of the 60-Hz field rate used to eliminate flicker. Some of this redundancy can be removed by techniques of conditional replenishment,[1–5] in which only picture elements that have changed by at least a threshold amount are transmitted. Conditional replenishment has recently been improved by displacement compensation.[6–13] The displacement compensation schemes have been developed primarily to compensate for translational displacement of objects in uniform illumination. This is done by estimating the translation of an object in the scene and using it for predictive coding by taking differences of elements with respect to appropriately displaced elements in the previous frame. Such schemes have been developed in

both the picture element (pel) domain[10,11] and the transform domain.[12,13]

In this paper, we extend the translational displacement model of the picture intensity and develop recursive algorithms for estimating the parameters associated with the extended model. The extended model incorporates spatial and temporal variations of illumination, as well as translational displacement of moving objects. Illumination acts as a multiplicative factor or gain on the reflectance of objects in the scene. The extended model thus has two parameters, gain and displacement. The coder estimates these two parameters recursively from the previously transmitted data so that no additional information need be transmitted to specify tbem.

Illumination and displacement variations occur to different degrees in different television scenes and, therefore, the efficiency of these new compensation schemes varies from scene to scene. Three scenes containing a sequence of 60 frames each bave been used to evaluate the efficiency of the new coding algorithms. These sequences contain distinctly different types of motion of objects and frame-to-frame variation of illumination. We summarize our findings as follows. Encoders using gain compensation (improved prediction and segmentation) alone require a slight increase in hardware compared to the conditional replenishment encoders but are capable of reducing the bit rate by about 11 to 51 percent, depending upon the scene. Displacement compensation encoders[10,11] are more complex and reduce the bit rate by 15 to 61 percent compared to the conditional replenishment. Gain and displacement compensation together reduce the bit rate by about 32 to 63 percent. These improvements in bit rates are a function of the type of scene. In one of the scenes, the gain and displacement compensation has reduced the peak bit rate by about 75 percent compared to conditional replenishment.

II. LUMINANCE VARIATION AND ITS COMPENSATION

Earlier work[10,11] introduced a recursive technique for estimating the translational displacement of objects moving in uniform illumination. The image intensity in the present frame at a spatial location $\mathbf{x}$ and time t was assumed to be equal to the spatially displaced intensity at time t-τ

$$I(\mathbf{x}, t) = I(\mathbf{x} - \mathbf{D}, t - \tau), \tag{1}$$

where $\mathbf{D}$ is the displacement of the object and τ is either the field or the frame interval. The steepest descent algorithm for recursive estimation of displacement is given by

$$\hat{\mathbf{D}}^{i} = \hat{\mathbf{D}}^{i-1} - \epsilon \cdot \mathrm{DFD}(\mathbf{x}, \hat{\mathbf{D}}^{i-1})\, \nabla I(\mathbf{x} - \hat{\mathbf{D}}^{i-1}, t - \tau), \tag{2}$$

where the displaced field or frame difference, $\mathrm{DFD}(\cdot,\cdot)$, is defined by

$$\text{DFD}(\mathbf{x}, \hat{\mathbf{D}}) = I(\mathbf{x}, t) - I(\mathbf{x} - \hat{\mathbf{D}}, t - \tau), \tag{3}$$

ϵ is a small positive constant influencing convergence rate, ∇I is the spatial gradient of the intensity, and $\hat{\mathbf{D}}^i$ is the ith estimate of $\mathbf{D}$.

In several situations, the above model of image intensity is not adequate. The three examples of this that follow involve certain conditions on illuminance $L(\mathbf{x}, t)$, and reflectance $R(\mathbf{x}, t)$ comprising the scene.

(*i*) Time modulation of illuminance, reflectance being constant with respect to time. This situation corresponds to shadows created in the background of a scene as a result of a moving object. It is modeled by

$$I(\mathbf{x}, t) = L(t)R(\mathbf{x}) \tag{4}$$

$$I(\mathbf{x}, t - \tau) = L(t - \tau)R(\mathbf{x}) \tag{5}$$

and, therefore,

$$I(\mathbf{x}, t) = \frac{L(t)}{L(t - \tau)} I(\mathbf{x}, t - \tau). \tag{6}$$

(*ii*) Translational displacement of reflectance due to object motion; spatially nonuniform but temporally constant illumination. This occurs quite commonly, since the illumination is generally not perfectly uniform. It is modeled by

$$I(\mathbf{x}, t) = L(\mathbf{x})R(\mathbf{x}, t) \tag{7}$$

$$I(\mathbf{x}, t - \tau) = L(\mathbf{x})R(\mathbf{x} + \mathbf{D}, t) \tag{8}$$

and, therefore,

$$I(\mathbf{x}, t) = \frac{L(\mathbf{x})}{L(\mathbf{x} - \mathbf{D})} I(\mathbf{x} - \mathbf{D}, t - \tau) \tag{9}$$

or, alternatively,

$$I(\mathbf{x}, t) = \frac{R(\mathbf{x}, t)}{R(\mathbf{x} + \mathbf{D}, t)} I(\mathbf{x}, t - \tau). \tag{10}$$

(*iii*) Translational displacement of illumination, spatially nonuniform but temporally constant reflectance. This is the dual of case (*ii*) and can be caused by a shadow created in the background by a moving object. It is modeled by

$$I(\mathbf{x}, t) = L(\mathbf{x}, t)R(\mathbf{x}) \tag{11}$$

$$I(\mathbf{x}, t - \tau) = L(\mathbf{x} + \mathbf{D}, t)R(\mathbf{x}) \tag{12}$$

and, therefore,

$$I(\mathbf{x}, t) = \frac{R(\mathbf{x})}{R(\mathbf{x} - \mathbf{D})} I(\mathbf{x} - \mathbf{D}, t - \tau) \tag{13}$$

or, alternatively,

$$I(\mathbf{x}, t) = \frac{L(\mathbf{x}, t)}{L(\mathbf{x} + \mathbf{D}, t)} I(\mathbf{x}, t - \tau). \tag{14}$$

Thus, in several common cases, a multiplicative factor, whicb may vary temporarily and spatially, describes the intensity changes. We are therefore motivated to generalize the model of eq. (1) to

$$I(\mathbf{x}, t) = \rho_1 I(\mathbf{x}, t - \tau) \tag{15a}$$

and

$$I(\mathbf{x}, t) = \rho_2 I(\mathbf{x} - \mathbf{D}, t - \tau). \tag{15b}$$

Equations (15a) and (15b) are alternative models of frame-to-frame intensity variation. However, they are not equivalent alternatives. In a typical television scene, different parts of the picture change in different ways. It is important to recognize these different parts and compensate the intensity changes by appropriately cbosing (15a) or (15b). We discuss an algorithm in the next section to identify these different parts and use appropriate compensation for them.

The compensation for the above variation of intensity can be accomplished by estimating ρ_1, ρ_2, and $\mathbf{D}$ using gradient-type algorithms as before. The algorithm for compensation of (15a) is:

$$\hat{\rho}_1^{i+1} = \hat{\rho}_1^i + \epsilon_1 [I(\mathbf{x}, t) - \hat{\rho}_1^i I(\mathbf{x}, t - \tau)] I(\mathbf{x}, t - \tau). \tag{16}$$

For compensation of (15b), both ρ_2 and $\mathbf{D}$ need to be estimated. This is accomplished by

$$\hat{\rho}_2^{i+1} = \hat{\rho}_2^i + \epsilon_1 \text{DFD}(\mathbf{x}, \hat{\rho}_2^i, \hat{\mathbf{D}}^i) I(\mathbf{x} - \hat{\mathbf{D}}^i, t - \tau) \tag{17}$$

$$\hat{\mathbf{D}}^{i+1} = \hat{\mathbf{D}}^i - \epsilon_2 \hat{\rho}_2^i \text{DFD}(\mathbf{x}, \hat{\rho}_2^i, \hat{\mathbf{D}}^i) \nabla I(\mathbf{x} - \hat{\mathbf{D}}^i, t - \tau), \tag{18}$$

where $\text{DFD}(\cdot, \cdot, \cdot)$ is defined by

$$\text{DFD}(\mathbf{x}, \hat{\rho}_2, \hat{\mathbf{D}}) = I(\mathbf{x}, t) - \hat{\rho}_2 I(\mathbf{x} - \hat{\mathbf{D}}, t - \tau) \tag{19}$$

and ϵ_1 and ϵ_2 are positive scalar constants. The iterations may proceed from sample to sample along a scan line. Use of algorithms (16) to (19) assumes that ρ_1, ρ_2, and $\mathbf{D}$ vary sufficiently slowly spatially—an assumption that appears to be often valid in practice.

We note in passing that eq. (15b) is the appropriate model for intensity variation due to translational motion of objects. For uniform illumination, ρ_2 will be unity and $\mathbf{D}$ will be equal to the displacement. However, we can see from (10) that eq. (15a) also provides a description of object motion. Because of this, eq. (15a), with $\hat{\rho}_i$ obtained from (16), in some cases approximates variations of intensity due to object translation. This occurs if the parameter ρ_1 varies sufficiently slowly to be learned by (16). An example of this is shown in Fig. 1, which

shows a vertical edge in intensity that is displaced from one frame to the next in the horizontal direction by two picture elements. Recurrence relationship (16) is carried out in Fig. 1, where a plot of $\hat{\rho}_1$ along the scan line is shown. Also shown is the frame difference and the gain-compensated frame difference *GFDIF*:

$$GFDIF(\mathbf{x}, \hat{\rho}_1, t) = I(\mathbf{x}, t) - \hat{\rho}_1 I(\mathbf{x}, t - \tau). \tag{20}$$

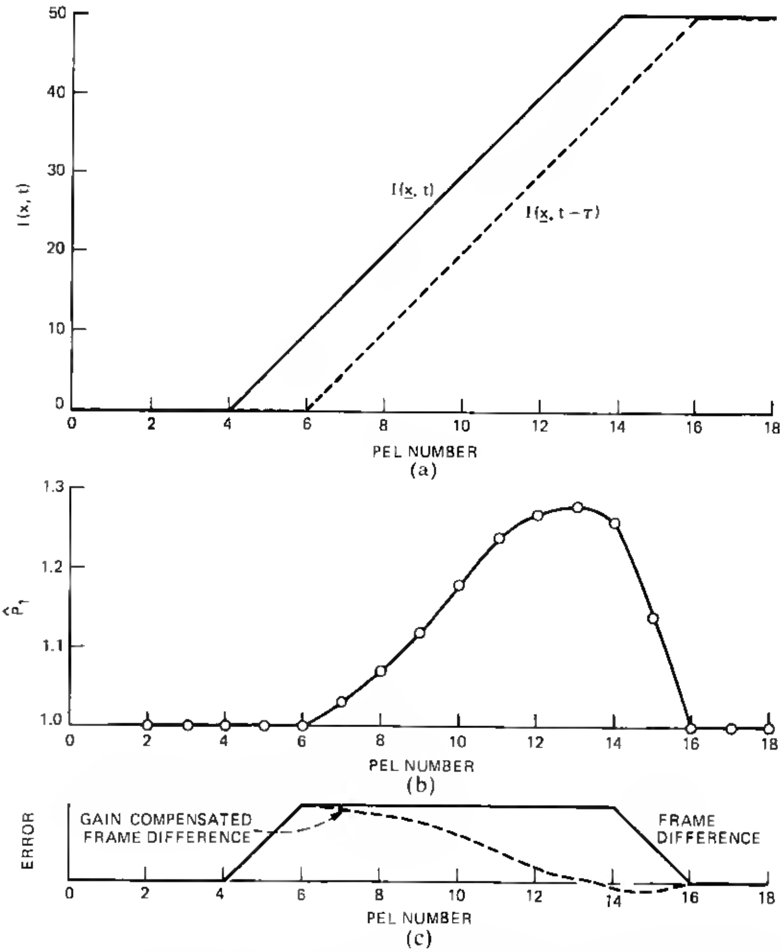

Fig. 1—(a) A synthetic vertical edge whose intensity for any scan line is plotted as a function of horizontal position for two consecutive frames. The horizontal shift in the edge is 2 pels per frame. (b) Plot of the value of $\hat{\rho}_1$ generated recursively from eq. (16), starting with a value of 1.0. Recursion is assumed to proceed pel by pel in the direction of scanning. (c) Plot of frame difference (or the prediction error in the conditional replenishment coders) and the gain-compensated frame difference (or the prediction error in the gain-compensated coders) as a function of pel position.

It is seen then that the gain-compensated frame difference is able to provide a certain degree of displacement compensation and, therefore, results in errors that are less than the frame differences traditionally used in conditional replenishment.

III. CODER SIMULATIONS

We have performed computer simulations of four different coders. Two of these, frame difference conditional replenishment and displacement compensation, were simulated for comparison purposes. The parameters used for these two were identical to those given in our earlier paper.[11] The third simulated coder (called gain-compensated coder) worked as follows:

Let the frame difference, *FDIF*, and the gain-compensated frame difference, *GFDIF*, be defined as

$$FDIF(\mathbf{x}, t) = I(\mathbf{x}, t) - I(\mathbf{x}, t - \tau). \tag{21}$$

$$GFDIF(\mathbf{x}, \rho_1, t) = I(\mathbf{x}, t) - \rho_1 I(\mathbf{x}, t - \tau). \tag{22}$$

Then the intensity at pel Z (Fig. 2) is predicted by:

$$P_Z = \begin{cases} I(\bar{Z}, t - \tau), & \text{if} \quad \sum_{\mathbf{x} \in \{B,C,D\}} |FDIF(\mathbf{x}, t)| \\ & \quad \leq \sum_{\mathbf{x} \in \{B,C,D\}} |GFDIF(\mathbf{x}, t, \hat{\rho}_1^C)|, \\ \hat{\rho}_1^C I(\bar{Z}, t - \tau), & \text{otherwise} \end{cases} \tag{23}$$

where $\hat{\rho}_1^C$ is the previously estimated $\hat{\rho}_1$ at location C. The iteration for ρ_1 proceeds along the scan line according to eq. (16). However, to simplify multiplications that are necessary for implementing eq. (16), the algorithm was modified as

$$\hat{\rho}_1^{i+1} = \hat{\rho}_1^i + \epsilon_1 \mathrm{sgn}[I(\mathbf{x}, t) - \hat{\rho}_1^i I(\mathbf{x}, t - \tau)], \tag{24}$$

where

$$\mathrm{sgn}(u) = \begin{cases} 0, & \text{if} \quad u = 0 \\ \dfrac{u}{|u|}, & \text{otherwise.} \end{cases} \tag{25}$$

In our simulations, $\hat{\rho}_1$ was constrained to be in the interval [15/16, 17/16], and ϵ_1 was taken to be 1/128. Iterations were carried out pel by pel in the direction of the scanning with the last value of $\hat{\rho}_1$ on a line being used as an initial estimate for the next line. This coder requires a simple modification of the frame difference conditional replenishment coder. A block diagram of the coder is shown in Fig. 3.

The fourth coder consists of gain and translation compensation used simultaneously. In this case, the prediction for the present pel Z of Fig.

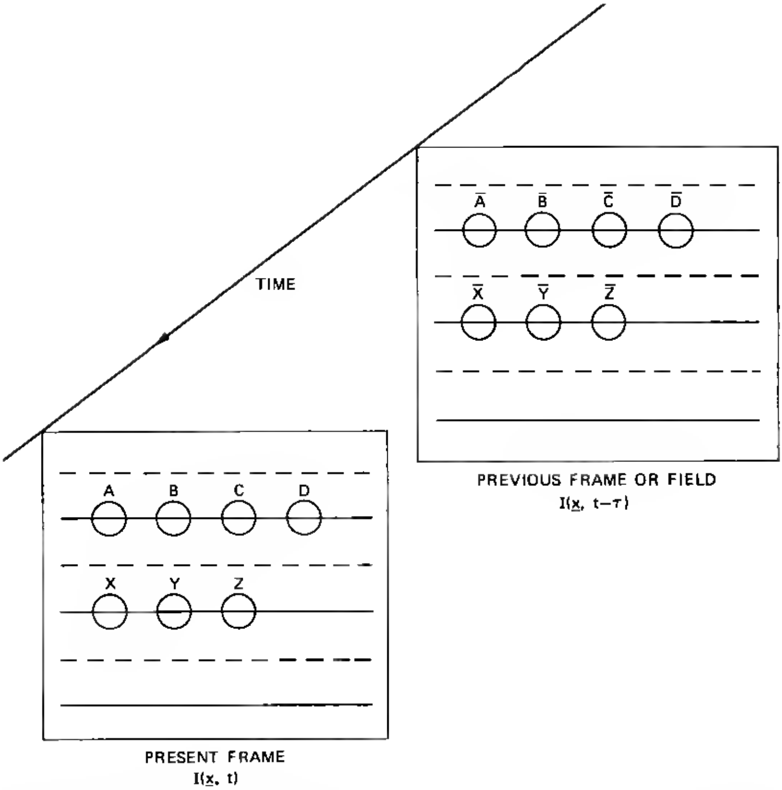

Fig. 2—Configuration of pels for gain compensation. Dashed and solid lines denote scan lines in alternate fields. $I(Z, t)$ is predicted by $\hat{\rho}_1^C I(\bar{Z}, t-\tau)$, where $\hat{\rho}_1^C$ is the estimate of $\hat{\rho}_1$ at pel position C.

2 is made by switching between the three predictors given below, where τ_1 and τ_2 are frame and field intervals, respectively.

$$P_1 = I(\bar{Z}, t-\tau_1)$$
$$P_2 = \hat{\rho}_1^C I(\bar{Z}, t-\tau_1)$$
$$P_3 = \hat{\rho}_2^C I(\bar{Z} - \hat{\mathbf{D}}^C, t-\tau_2), \tag{26}$$

where $\hat{\mathbf{D}}^C$, $\hat{\rho}_2^C$ and $\hat{\rho}_1^C$ are the estimated translation and gain for pel C from the previous line. $\hat{\rho}_1^C$ is estimated recursively according to eq. (24) with $\tau = \tau_1$. $\hat{\rho}_2^C$, $\hat{\mathbf{D}}^C$ are estimated using a simplification of eq. (17) and (18):

$$\hat{\rho}_2^{i+1} = \hat{\rho}_2^i + \epsilon_1 \operatorname{sgn}[\text{DFD}(\mathbf{x}, \hat{\rho}_2^i, \hat{\mathbf{D}}^i)] \tag{27}$$
$$\hat{\mathbf{D}}^{i+1} = \hat{\mathbf{D}}^i - \epsilon_2 \operatorname{sgn}[\text{DFD}(\mathbf{x}, \hat{\rho}_2^i, \hat{\mathbf{D}}^i)] \cdot \operatorname{sgn}[\nabla I(\mathbf{x} - \hat{\mathbf{D}}^i, t-\tau)],$$

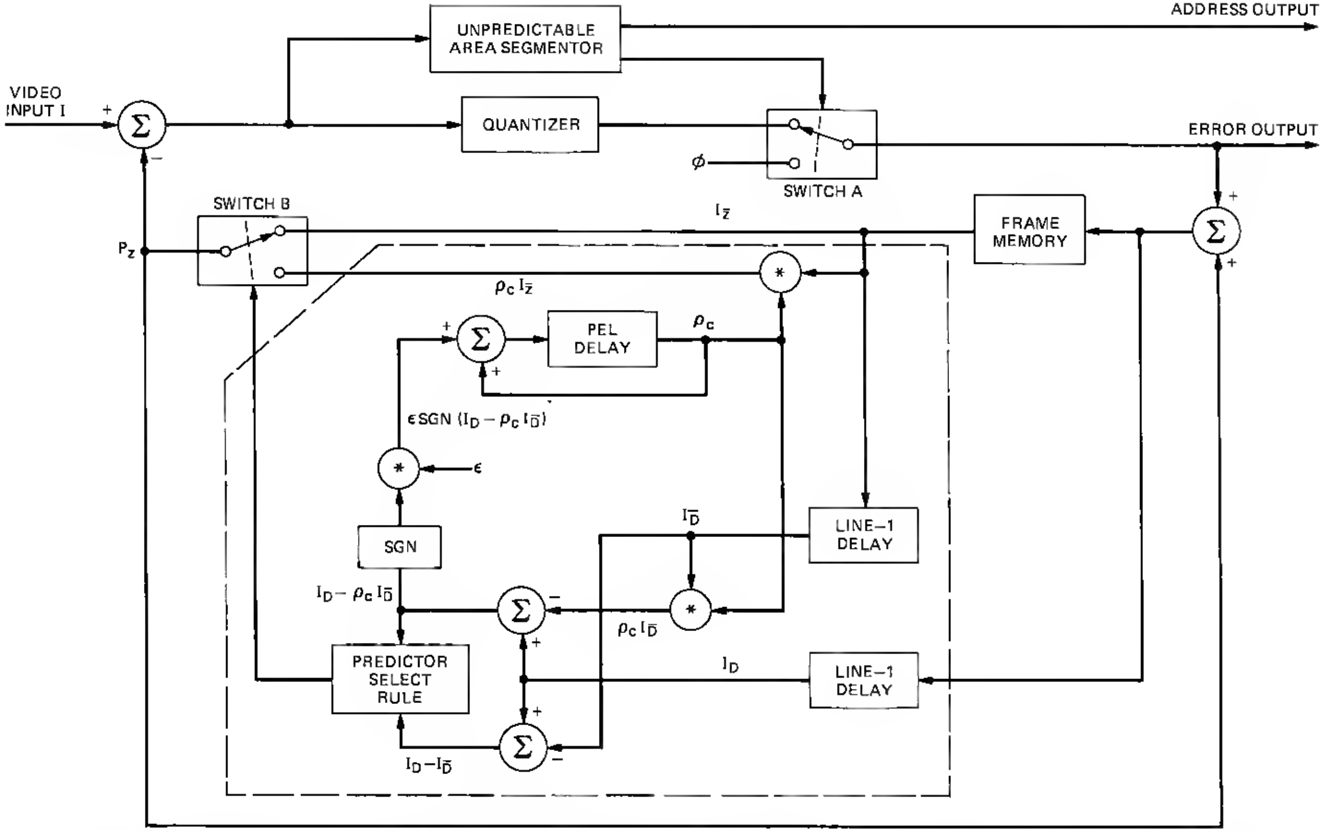

Fig. 3—Block diagram of a gain-compensated coder. Without the dotted section, the DPCM loop of a frame-difference conditional replenishment encoder is implemented. Thus, gain compensation requires a simple modification of the conditional replenishment coder.

Table I—Average bits per field

Average bits are computed by averaging over 60 frames of the sequences Judy, Mike and Nadine, and Mike and John, for the four coders. The numbers for conditional replenishment and displacement compensation are taken from Refs. 10 and 11.

Coder Scene	Conditional Replenishment	Displacement Compensation	Gain Compensation	Gain and Displacement Compensation
Judy	38,576	15,044	19,012	14,130
Mike and Nadine	94,975	71,407	84,401	64,401
Mike and John	68,136	56,548	40,164	36,612

where $\tau = \tau_2$. As in the gain-compensated coder, initial estimates of $\hat{\rho}_1$, $\hat{\rho}_2$, and $\hat{\mathbf{D}}$ of the beginning of a line were taken to be the corresponding estimates at the last pel of the previous line. ϵ_1 and ϵ_2 are taken to be 1/128, and 1/16, respectively. Iteration proceeds pel by pel in the direction of scanning. The selection of the predictor for a pel from the above three predictors is made by choosing the one that resulted in the least error for the adjacent previous line elements. Referring to Fig. 2, the selection rule is as follows:

$$P^Z = \begin{cases} P_1 & ; \text{ if } \quad E_1 = MIN(E_1, E_2, E_3), \\ P_2 & ; \text{ if } \quad E_2 = MIN(E_1, E_2, E_3) \text{ and } (E_1 \neq E_2), \\ P_3 & ; \text{ otherwise,} \end{cases} \tag{29}$$

where

$$E_1 = \sum_{\mathbf{x} \epsilon (B,C,D)} |FDIF(\mathbf{x}, t)|$$

$$E_2 = \sum_{\mathbf{x} \epsilon (B,C,D)} |GFDIF(\mathbf{x}, t, \hat{\rho}_1^C)|$$

$$E_3 = \sum_{\mathbf{x} \epsilon (B,C,D)} |\mathrm{DFD}(\mathbf{x}, \hat{\rho}_2^C, \mathbf{D}^C)|$$

and $MIN(\cdot, \cdot, \cdot)$ is the minimum value of its three variables. Using the techniques described in Ref. 11, the computation of E_1, E_2, and E_3 can be significantly simplified by substitution of other *FDIF*, *GFDIF*, and DFD signals.

Segmentation of pels into those selected for transmission and those dropped from transmission for the first coder is described in Ref. 10 and 11. For coders 2, 3, and 4, simple segmentation was used by thresholding* the magnitude of the prediction error: whenever the magnitude of the prediction error exceeded a threshold (3 out of 256), prediction error was sent in quantized form; otherwise, it was dropped from transmission and the pel was reconstructed at the receiver

* This type of segmentation is different from that used in coder 1 (conditional replenishment) where an attempt is made to form contiguous areas of transmitted pels. This increases the number of transmitted pels and consequently bits/field, especially if run-length coding is used for addressing. A significant part of the reduction in bit rate by gain compensation is due to the different segmentation used.

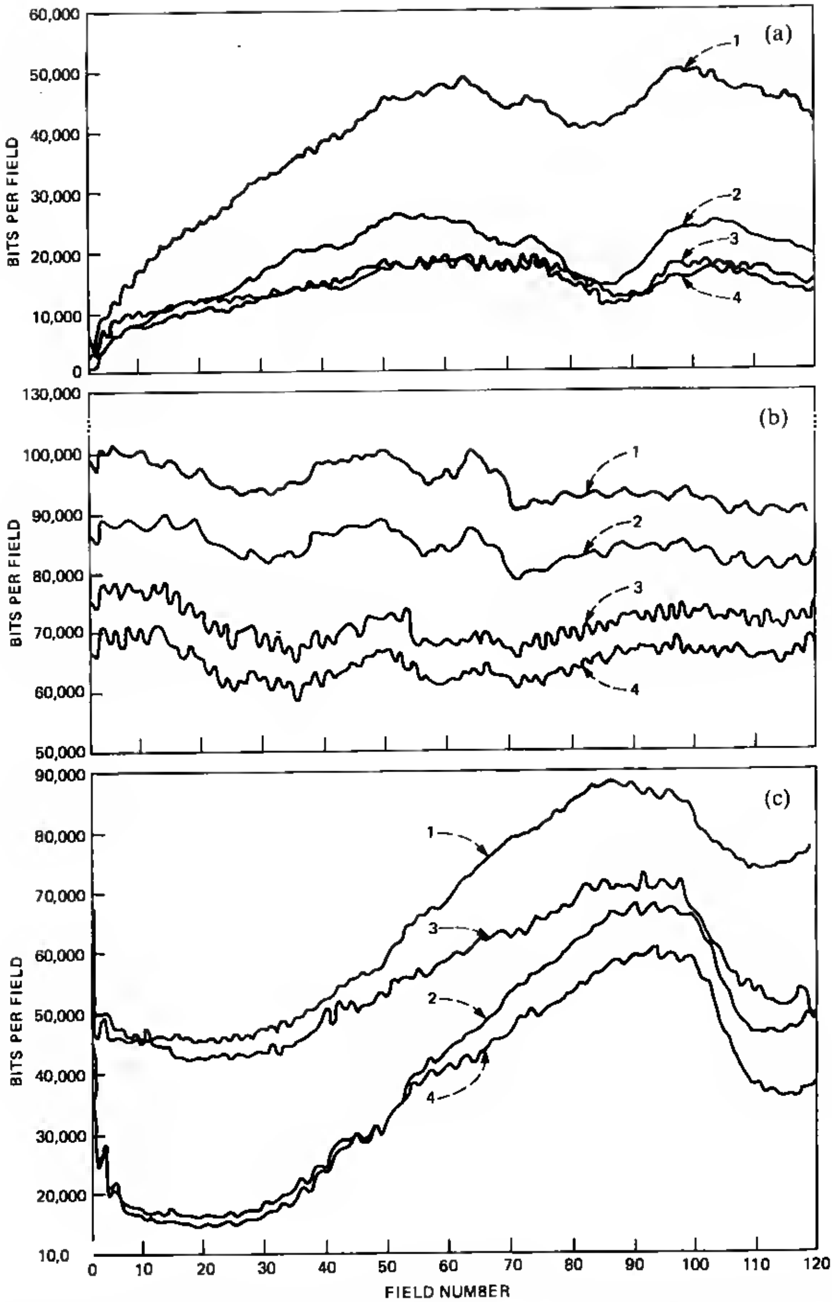

Fig. 4—Plots of bits/field versus the field number using the four coders for the scenes (a) Judy, (b) Mike and Nadine, and (c) Mike and John. The four coders are: (1) frame difference conditional replenishment; (2) gain compensation; (3) displacement compensation; (4) gain and displacement compensation.

assuming zero prediction error. Thresholds were adjusted to give a good quality picture in which coding distortions were visible but not annoying. The quantizer used for all the four coders was 35-level, symmetric and companded. It is shown in Fig. 10 of Ref. 10. Bit rates were computed by adding the bits necessary to specify the prediction error and their addresses. Prediction error bits were approximated by multiplying the entropy of the transmitted prediction error by the number of unpredictable pels. The addressing bits were calculated by standard run length coding (in the direction of the scan line) of the unpredictable pels.

Figures 4a, 4b, and 4c show plots of bit rates with respect to time for three 60-frame sequences. The first two sequences, Judy and Mike and Nadine, are the same as those used in Ref. 10. The third sequence, Mike and John, contains large areas of nonuniform illumination, movement of shadows, and the people entering the camera field of view (25th frame) and walking briskly around each other. Although there are no moving objects in frames 0 to 24, the luminance of a considerable area changes as a result of shadows generated by objects out of camera view. It should be noted that the second (Mike and Nadine) and third sequences are similar; however, Mike and Nadine is a panned view of two people who are always in the camera view. The percent of moving area (as determined by the "frame difference"

Fig. 5—One frame from the scene sequence Mike and John. The speed of each person is approximately 5 pels per frame.

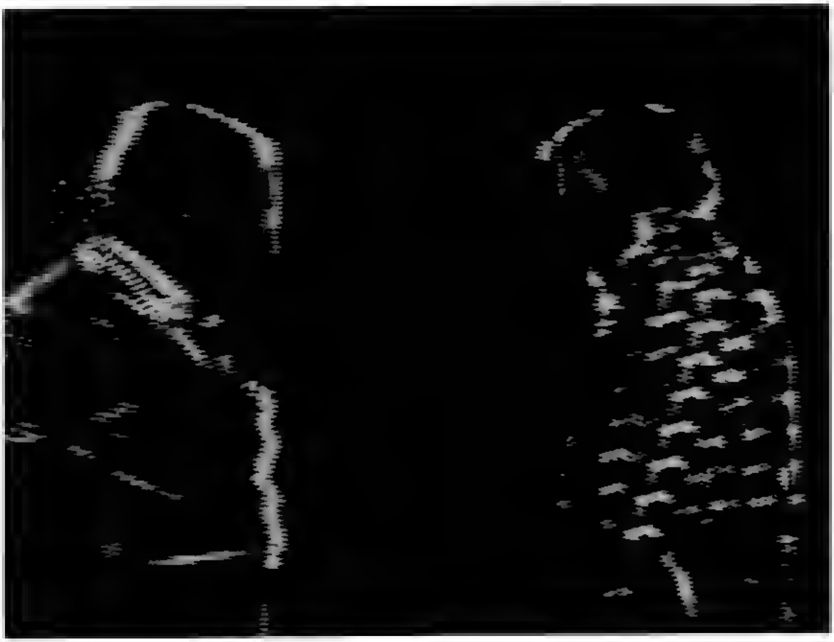

Fig. 6—Frame difference signal for the frame of Fig. 5. Time-varying illumination results in changing "background" intensities in the central region of the frame.

segmentor of coder 1) for the Mike and John sequence varies between 53 and 84. Figures 4a, 4b, and 4c has each four curves for the four coders. Parameters of each of these four coders were adjusted such that the picture quality was approximately equivalent as indicated by informal subjective tests made by the authors. Also Table I shows the average bit rates (over 60 frames) for the four coders and three scenes.

For the (head and shoulders) scene Judy, where the illumination is close to uniform, the gain-compensated coder results in average bit rates that are about 50 percent below those of the frame difference conditional replenishment coder. The displacement-compensated coder, on the other hand, results in about 61 percent decrease. Thus, gain compensation reduces the coder bit rate by a significant amount without the increase in the complexity associated with the displacement-compensated encoder. The gain- and displacement-compensated encoder reduce the bit rate by 63.4 percent for Judy. This additional decrease due to gain compensation is small, perhaps, as a result of relatively uniform illumination and lack of shadows in the scene. For the scene Mike and Nadine, again, most of the decrease in bit rate is provided by displacement-compensation and gain-compensation results in bit rates that are between the conditional replenishment and

displacement compensation. Also, combining gain and displacement compensation provides an additional decrease of 7.3 percent.

The scene Mike and John contains large areas of nonuniform illumination and moving shadows. The first 25 frames of this sequence contain no object motion. Most of the changes in the intensity are a result of the moving shadows cast by objects not in the camera view. As a result, the bit rates of the conditional replenishment and displacement compensated coder are similar for the first 25 frames and the gain compensation reduces the bit rates significantly. Following the 25th frame, moving objects enter the camera view, and displacement compensation is clearly superior to conditional replenishment. Even here, however, gain compensation performs slightly better than displacement compensation because of the large changes in illumination still taking place. Figures 5, 6, and 7 show the input scene, the frame difference, and the regions of different predictor usage, respectively, for one of the frames of the sequence Mike and John processed by the gain- and displacement-compensated coder. It is seen that, in most of the areas of shadows, the gain-compensation predictor is used, whereas in areas of pure motion the displacement-compensated predictor is used.

Fig. 7—Regions of different predictor use by the gain- and displacement-compensated coder for the frame of Fig. 5. Bright pels correspond to displacement-compensated predictor [P_3 of eq. (26)], light pels correspond to gain-compensated predictor [P_2 of eq. (26)], and dark pels correspond to previous frame predictor [P_1 of eq. (26)]. It is seen that unchanging background areas use the previous frame predictor, and most of the shadow regions use the gain-compensated predictor.

IV. CONCLUSIONS

We have presented algorithms for estimating parameters associated with certain common types of frame to frame intensity variations. The parameters, gain and displacement, are estimated by recursive algorithms using information previously transmitted by the coder. Using the estimates of these parameters, adaptive predictors and a segmentor to determine which pels need to be transmitted are synthesized for an interframe DPCM coder. Computer simulations using three scenes containing 60 frames each indicate that, compared to conditional replenishment, the decrease in bit rates using gain compensation is between 11 and 51 percent; using displacement compensation is between 15 and 61 percent, and using gain and displacement is between 32 and 63 percent. These decreases are a function of the type of intensity variation in a scene, but may be typical for many videoconferencing and broadcasting applications.

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